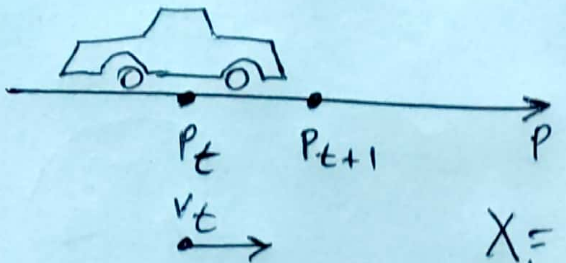
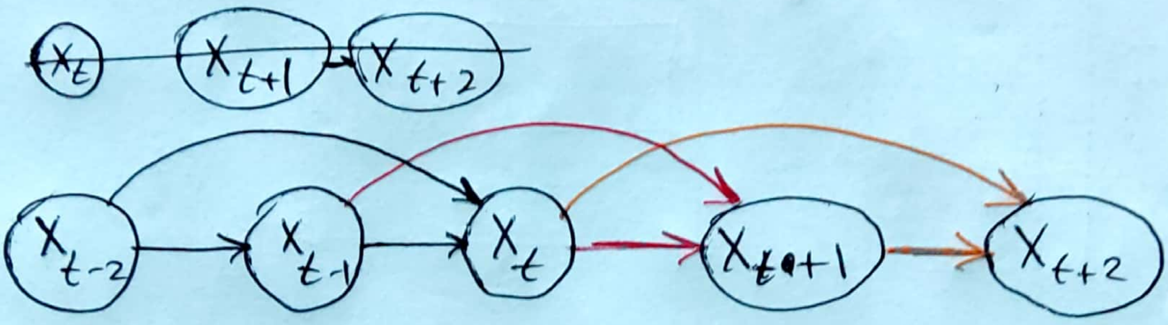


$$X_t = \begin{bmatrix} p_t \\ v_t \end{bmatrix} \qquad X_t = \begin{bmatrix} p_t \\ v_t \end{bmatrix} = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} p_{t-1} \\ v_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_p \\ \varepsilon_v \end{bmatrix}$$

$$X_t = A X_{t-1} + \varepsilon_x$$

$$\varepsilon_x \sim \cancel{A} \, P_{\varepsilon_x}$$

$$P(X_t \mid X_{t-1}) = P_\varepsilon (X_t - A X_{t-1})$$



Solution 2

$$\vec{X}_t = \begin{bmatrix} p_t \\ v_t \end{bmatrix} \Longrightarrow \vec{X}_t = \begin{bmatrix} p_t \\ v_t \\ a_t \end{bmatrix}$$

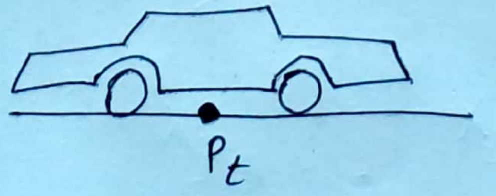
$$\vec{X}_t = \begin{bmatrix} p_t \\ v_t \\ a_t \end{bmatrix} = \begin{bmatrix} p_{t-1} + \Delta t \, v_{t-1} + \frac{1}{2} \Delta t^2 a_{t-1} + \varepsilon_p \\ v_{t-1} + \Delta t \, a_{t-1} + \varepsilon_v \\ a_{t-1} + \varepsilon_a \end{bmatrix}$$

$$\begin{bmatrix} p_t \\ v_t \\ a_t \end{bmatrix} = \begin{bmatrix} 1 & \Delta t & \frac{\Delta t^2}{2} \\ \emptyset & 1 & \Delta t \\ \emptyset & \emptyset & 1 \end{bmatrix} \begin{bmatrix} p_{t-1} \\ v_{t-1} \\ a_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_p \\ \varepsilon_v \\ \varepsilon_a \end{bmatrix}$$

$$\vec{X}_t = A \, \vec{X}_{t-1} + \vec{\varepsilon}_x$$

$$\begin{array}{c} X_t \\ \downarrow \\ O_t \end{array}$$

O_t gps signal



$$O_t = g_t = P_t + \varepsilon_g = \underbrace{\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}}_{B} \begin{bmatrix} P_t \\ v_t \\ a_t \end{bmatrix} + \varepsilon_g$$

g_t → gps ε_g → gps noise

$$O_t = B X_t + \varepsilon_0$$

$$P(O_t | X_t) = P_{\varepsilon_0}(O_t - B X_t)$$

$$O_t = \begin{bmatrix} gps \\ acc \end{bmatrix} = \begin{bmatrix} g_t \\ a'_t \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} P_t \\ V_t \\ a_t \end{bmatrix} + \begin{bmatrix} \varepsilon_g \\ \varepsilon_{a'} \end{bmatrix}$$

acc → شتاب سنج $\varepsilon_{a'}$ → نویز شتاب سنج

$$O_t = B X_t + \varepsilon_0$$

$$P(X_t | O_1 — O_{t-1}) = \frac{P(X_t, O_1 — O_{t-1})}{\sum_{X'_t} P(X'_t, O_1 — O_{t-1})}$$

$$= \frac{\sum_{X_{t-1}} \cdots \sum_{X_2} \sum_{X_1} P(X_1 — X_t, O_1 — O_{t-1})}{D}$$

$$= \frac{\sum_{X_{t-1}} \cdots \sum_{X_2} \sum_{X_1} P(X_2|X_1) P(X_3|X_2) \cdots P(X_t|X_{t-1}) P(O_1|X_1) \cdots P(O_{t-1}|X_{t-1})}{D}$$

$$\text{pred}(X_t) = P(X_t \mid O_1, O_2, \dots, O_{t-1})$$

$$\text{corr}(X_t) = P(X_t \mid O_1, O_2, \dots O_t)$$

$$\text{pred}(X_t) = P(X_t \mid O_1, \dots, O_{t-1}) = \overset{\int}{\sum_{X_{t-1}}} P(X_t, X_{t-1} \mid O_1 — O_{t-1})$$

$$= \sum_{X_{t-1}} P(X_t \mid X_{t-1}, O_1, O_2, \dots, O_{t-1}) \, P(X_{t-1} \mid O_1 — O_{t-1})$$

$$= \sum_{X_{t-1}} P(X_t \mid X_{t-1}) \, \text{corr}(X_{t-1})$$

continuous

$$\text{pred}(X_t) = \int p(X_t \mid X_{t-1}) \, \text{corr}(X_{t-1}) \, dX_{t-1}$$

$$\text{corr}(X_t) = P(X_t \mid O_1 — O_t) = P(X_t \mid O_1, —, O_{t-1}, O_t)$$

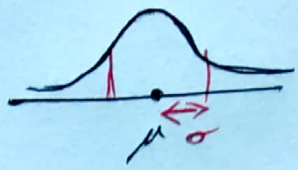
$$= \frac{P(O_t \mid X_t, O_1 — O_{t-1}) \, P(X_t \mid O_1 — O_{t-1})}{P(O_t \mid O_1 — O_{t-1})}$$

$$= \frac{P(O_t \mid X_t, O_1 — O_{t-1}) \, P(X_t \mid O_1 — O_{t-1})}{\sum_{X'_t} P(O_t, X'_t \mid O_1 — O_{t-1})}$$

$$= \frac{P(O_t \mid X_t, O_1 — O_{t-1}) \, p(X_t \mid O_1 — O_{t-1})}{\sum_{X'_t} P(O_t \mid X'_t, O_1 — O_{t-1}) \, P(X'_t \mid O_1 — O_{t-1})}$$

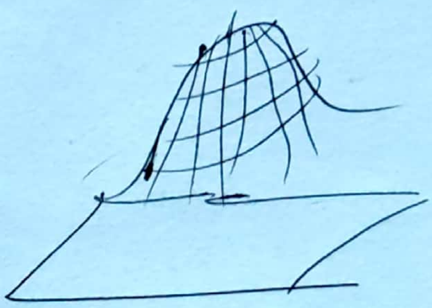
$$= \frac{P(O_t \mid X_t) \, \text{pred}(X_t)}{\sum_{X'_t} P(O_t \mid X'_t) \, \text{pred}(X'_t)}$$

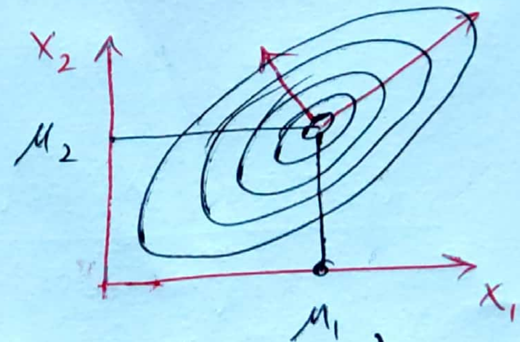
$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(\frac{(x-\mu)^2}{\sigma^2} \right)$$



$$p\left(\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \right) = \frac{1}{(2\pi)^{n/2} |\Sigma|^{1/2}} \exp\left((x-\vec{\mu})^T \Sigma^{-1} (x-\mu) \right)$$

$$\mu = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_n \end{bmatrix} \qquad \Sigma = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13} & \cdots \\ \sigma_{21} & \sigma_2^2 & & \\ \sigma_{31} & & \ddots & \\ \vdots & & & \sigma_n^2 \end{bmatrix}$$





$$p(X_t | X_{t-1}) = \text{Normal}(X_t - AX_{t-1}; \emptyset, \Sigma_x)$$

$$= \frac{1}{(2\pi)^{n/2} |\Sigma_x|^{1/2}} \exp\left((X_t - AX_{t-1})^T \Sigma_x^{-1} (X_t - AX_{t-1}) \right)$$

observation model

$$p(O_t | X_t) = \text{Normal}(O_t - BX_t, \emptyset, \Sigma_o)$$

$$= \frac{1}{(2\pi)^{n/2} |\Sigma_o|^{1/2}} \exp\left((O_t - BX_t)^T \Sigma_o^{-1} (O_t - BX_t) \right)$$